Abstract of Plenary Lectures

Laurent polynomial fibration over a discrete valuation ring and structure of an algebra which is locally Laurent polynomial algebra over an affine domain

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Let A be an affine domain over a field containing an affine domain R such that for every maximal ideal m of R, $A \otimes_R R_m$ is a polynomial algebra in n variables over the local ring R_m. In this set up a result of Bass-Connell-Wright says that A is a symmetric algebra of a finitely generated projective R-module of rank n. In my talk I will discuss the Laurent polynomial analogue of this famous local-global B-C-W theorem. Now suppose R is a discrete valuation ring containing the field of rationals and A is a finitely generated flat R-algebra whose generic as well as closed fibre is a polynomial algebra in n variables over respective fields. For $n > 2$, it is still an open problem whether A is a polynomial algebra over R. I will also discuss Laurent polynomial analogue of this question and results obtained. This talk is based on my joint work with Neena Gupta (ISI, Kolkata).

Polynomial maps with invertible sums of Jacobian matrices

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Let $F = X + H : \mathbb{C}^n \to \mathbb{C}^n$ be a polynomial map with the degree $d \geq 2$. Invertibility of a polynomial map F has been examined by several authors under certain conditions on the evaluated Jacobian matrices $JF(\alpha),\ \alpha \in \mathbb{C}^n$, for example, JH is strongly nilpotent, additive-nilpotent, or $JF(\alpha) + JF(\beta)$ is invertible.

In this talk, we discuss invertibility of a polynomial map F under the condition that $JF(\alpha_1) + \cdots + JF(\alpha_s)$ is invertible. We get invertibility of F when $s = d - 1$ or n, where d and n are the degree and the dimension of F, respectively. Especially, when $s = n$, F is characterized as a composition of an invertible linear map and an invertible polynomial map $X + H$ with linear part X, such that the subspace generated by $\{JH(\alpha) | \alpha \in \mathbb{C}^n\}$ consists of nilpotent matrices. This is a joint work with Hongbo Guo, M de Bondt, and Xiaosong Sun.

An approach to Zariski-Lipman Conjecture via Open Algebraic Surface Theory

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We will consider the following global version of the well-known conjecture of Zariski and Lipman about free module of derivations.

Conjecture. Let X be a complex normal quasi-projective surface. If the algebraic tangent bundle of X-Sing X is trivial then X is smooth.

We will verify the conjecture if X is affine and such that the log Kodaira dimension of X-Sing X is at most one. We will also make several interesting observations when X is projective. For the proof, we will use the theory of non-complete algebraic surfaces developed by Iitaka, Kawamata, Miyanishi, Fujita, Sakai, Sugie, Tsunoda, and other Japanese mathematicians. This is a joint work with Sagar Kolte.

Resolution of pencils and an application of minimal model program in threefolds

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One of the useful ways to investigate the structure of an affine algebraic variety which is realized as the complement of a hypersurface $S \subseteq \mathbb{P}^n$ lies in the observation of a suitable linear pencil Λ on $\mathbb{P}^n$ containing S as a member. This attempt is satisfactorily established in case of dimension two by the work due to Miyanishi and Sugie since the process of the resolution of base points of Λ and an application of an MMP can be performed in an explicit manner for surfaces. In this talk, we shall try to generalize such a consideration in dimension two to the case of threefolds. In particular, we exhibit concretely the process of the resolution of a linear pencil and that of MMP after the resolution in the case where $S \subseteq \mathbb{P}^3$ are smooth conics to analyze properties on $\mathbb{P}^3 \backslash S$.

Logarithmic plurigenera of some normal affine surfaces

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In this talk, I will give some results on logarithmic plurigenera of normal affine surfaces in any characteristic. Let S be a normal affine surface and S_0 its smooth locus. The results presented in this talk are as follows:

1. Suppose that the logarithmic Kodaira dimension of S_0 is one. Then its logarithmic 24th genus is positive. Moreover, if S is smooth, then its logarithmic bigenus is positive.

2. Suppose that S is not a rational surface. Then S_0 has non-negative logarithmic Kodaira dimension if and only if its logarithmic 12th genus is positive.

The Shestakov-Umirbaev inequality over a field of an arbitrary characteristic

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In 2003, Shestakov-Umirbaev solved the famous Nagata conjecture on the wildness of polynomial automorphisms. In their theory, a certain inequality for estimating the degrees of polynomials played an important role. The theory of Shestakov-Umirbaev was constructed in the case where the ground field is of characteristic zero. It is a difficult open problem to extend the theory to the case of positive characteristic, and the Nagata conjecture also remains open in the positive characteristic case. The crucial difficulty is the generalization of the above mentioned inequality. In this talk, we present a new idea for generalizing the inequality to the case of an arbitrary characteristic. Using our result, we derive the Jung-van der Kulk theorem of polynomial automorphisms in two variables.

Two-dimensional representations of finitely generated free group over an arbitrary field

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Let G be a free group with basis $\{u_1, u_2, \ldots, u_n\}$ and $\mathbb{F}$ an arbitrary field. For a two-dimensional linear representation of G over $\mathbb{F}$, let $g_i = \rho(u_i)$ for $i = 1, 2, \ldots, n$, $g_{ij} = \rho(u_i u_j) - 1$ for $1 \leq i < j \leq n$, $t_i = tr(g_i)$, $t_{ij} = tr(g_{ij})$, $e_i = det(g_i)$ and $e_{ij} = det(g_{ij})$. The first main result (Theorem 1) of this paper is a characterization for the existence of a two-dimensional representation ρ with preassigned (t_i, t_{ij}, e_i), $e_i \neq 0$. The second main result (Theorem 2) addresses the reducibility of such a representation. A classification of all reducible representations with given traces t_i, t_{ij} and determinants e_i is also given. These results extend the earlier work of L. N. Vaserstein and E. Whel in 2000 as well that of Z. B. Yan and H. You in 2003.

Vector fields on algebraic varieties

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Let X be a smooth algebraic variety defined over $\mathbb{C}$. A vector field v is a global section of the tangent bundle of X. If X is affine, it corresponds to a derivation D on the coordinate ring B. One expects that conditions on v or on D restrict the structure of X. In the talk, I will discuss on the following topics.

1. General features, e.g. rationality of Ker D when so is X.
2. Case where D is algebraic, i.e., locally finite.
3. Case where D is surjective. There is a famous theorem of Cerveau concerning the determination of a surjective derivation on $\mathbb{C}[x, y]$ which is proved analytically. We give a partial proof of it by means of algebro-geometric methods and try its generalization.

A surmise of Suslin on projective modules over an affine algebra

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We describe some recent progress on a question of Suslin raised in his Helsinki invited talk in 1978 on the freeness of stably free projective modules of rank $(d-1)$ over an affine algebra of dimension d over an algebraically closed field. This is joint work with Jean Fasel and R.G. Swan.

Poisson algebras, Weyl algebras and Jacobi pairs

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In this talk, we discuss the natural Poisson algebra structure $(P, [\cdot,\cdot], \cdot)$ on the space $P = C[y]((x^{-1/N}))$ for some sufficient large N, and introduce some automorphisms of P which are products of the automorphisms of forms e^{ad_H}. These automorphisms are used to study some properties of Jacobi pairs in P. In particular, starting from a Jacobi pair (F, G) in $C[x, y]$ which violates the two-dimensional Jacobian conjecture, by applying some variable changes, we obtain a pair still denoted by (F, G) in $C[x^{\pm 1/N}, y]$ with the form $F = x^{m/(m+n)}(f + F_0)$, $G = x^{n/(m+n)}(g + G_0)$ for some positive integers m, n, and f, g in $C[y]$, F_0, G_0 in $x^{-1/N}C[x^{-1/N}, y]$, such that F, G satisfy some additional conditions. We also generalize some results to the Weyl algebra $W = C[v]((u^{-1/N}))$ with relation $[u, v] = 1$, and obtain some properties of pairs (F, G) satisfying $[F, G] = 1$, referred to as Dixmier pairs.

On tame automorphisms of polynomial algebras

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In this talk, we discuss several problems on tame automorphisms of polynomial algebras. (1) Derksen showed that every tame automorphism in dimension $n \geq 3$ is a composition of affine ones and quadratic ones. It is still open whether every quadratic automorphism is tame. We introduce the so-called "additive-nilpotent" polynomial map, which generalizes the quadratic one. A polynomial map $X + H$ is called "additive-nilpotent" if the subspace spanned by $\{JH(\alpha)|\alpha \in \mathbb{C}^n\}$ is nilpotent. We show that such a map is invertible, and show that it is tame in dimension $n \leq 4$. (2) The co-invariant $c(\phi)$ of a polynomial map ϕ was introduced by Le Bryun, which is defined to be the dimension of the minimal sub-coalgebra of the Hopf algebra $\mathbb{C}[\mathbb{G}_a^n] = \mathbb{C}[x_1, \ldots, x_n]$ containing the $\phi(x_i)$'s. Up to now, little is known about the relation between the co-invariant and the tameness of a polynomial automorphism. We discuss the structure of Keller maps with small co-invariants, and show that a Keller map (resp. a quadratic one) ϕ in dimension n is a tame automorphism if $co(\phi) \leq n + 2$ (resp. $co(\phi) \leq n + 5$).

The embedding conjecture, the lifting problem and Tietze transformations

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The speaker will report some recent progress of the Lifting problem and Stable-tameness conjecture, as well as the connection with the Embedding conjecture, and a possible approach by the Tietze transformation.